ESTIMATING THE DISTORTION OF A TEMPERATURE FIELD BY FLAT TEMPERATURE AND THERMAL FLUX PROBES

B. A. Arutyunov, M. V. Belostotskii, and R. D. Stepanov

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Formulas are derived for calculating the error in temperature measurements which results from a distortion of the temperature field by temperature or thermal flux probes.

During the measurement of thermophysical properties of a substance by transient methods, the temperature field becomes distorted by temperature and thermal flux probes, as a consequence of their thermal conductivity and specific heat being different from those of the test specimen. These distortions affect the accuracy of both temperature and thermal flux measurements, and thus also the accuracy of the experiment. In this study here we will estimate such an error in the method of a momentary heat source, when the thermophysical properties of poor heat conductors are measured with flat metallic probes for sensing either the temperature or the thermal flux.

The problem is formulated as follows. In a specimen which simulates an infinitely large medium or an infinitely long rod with adiabatic conditions around the lateral surface and with the thermophysical properties λ_1, c_1, γ_1, there is generated at time zero a quantity of heat Q_1 in the plane 1—1 (Fig. 1). At a distance R there is located a temperature or a thermal flux probe of thickness Δ and with the thermophysical properties λ_2, c_2, γ_2. Without any probe, the temperature at distance R is defined by the relation:

$$T(0,\ \tau)_{\text{un}} = \frac{Q_1}{2c_1\gamma_1\sqrt{\pi a_1\tau}}\ \exp\left(-\frac{R^2}{4a_1\tau}\right), \tag{1}$$

where $a_1 = \lambda_1/c_1\gamma_1$.

The temperature is measured at time $\tau_{\max} = R_2/2a_1$ [1]. Then

$$T(0,\ \tau)_{\text{un}} = \frac{Q_1}{2c_1\gamma_1 R\sqrt{\frac{1}{2}\pi e}}.$$

We will now determine by how much $T(0,\ \tau)_{\text{un}}$ differs from the temperature read by the probe. An assumption will be made which simplifies the solution. Inasmuch as the thermal conductivity of poor heat conductors differs from that of the probe material (usually a metal) by factor of the order of 10^4, we let

$$\lambda/\lambda_1 \approx \infty.$$

The equation describing this problem can be written as

$$\frac{\partial T_1}{\partial\tau}(x,\ \tau) = a_1\frac{\partial^2 T_1}{\partial x^2}, \qquad 0 \leqslant x \leqslant \infty, \tag{2}$$

$$\frac{\partial T_3}{\partial\tau}(x,\ \tau) = a_1\frac{\partial^2 T_3}{\partial x^2}, \qquad -\infty \leqslant x \leqslant -\Delta. \tag{3}$$

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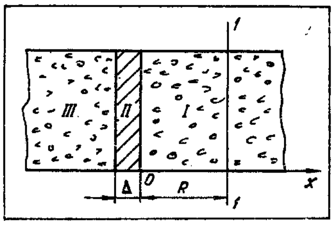

Fig. 1. Model of a temperature field for the problem of field distortion by temperature and thermal flux probes: III and I are the medium with the thermophysical properties to be determined, II is a thermal flux or a temperature probe, 1—1 is the plane in which a momentary heat pulse is generated.

The equation of heat balance for the probe yields

$$\lambda_1\left[\left(\frac{\partial T_3}{\partial x}\right)_{-\Delta}-\left(\frac{\partial T_1}{\partial x}\right)_0\right]=c_2\gamma_2\Delta\,\frac{\partial T_2}{\partial \tau}. \tag{4}$$

The initial conditions are

$$T_1(x,\ 0)=T_2(x,\ 0)=T_3(x,\ 0)=0. \tag{5}$$

The boundary conditions are

$$T_1(+\infty,\ \tau)=0, \tag{6}$$

$$T_3(-\infty,\ \tau)=0, \tag{7}$$

$$T_1(0,\ \tau)=T_2(\tau)=T_3(-\Delta,\ \tau). \tag{8}$$

This system of equations is solved by the method of the Laplace integral transformation

$$T_{L_3}(x,\ \tau)=\frac{Q_1}{c_1\gamma_1 k\Delta}\cdot\frac{\exp\left[-\sqrt{\frac{s}{a_1}}(R-\Delta-x)\right]}{\sqrt{s}\left(\sqrt{s}-\frac{2\sqrt{a_1}}{k\Delta}\right)}. \tag{9}$$

The original function of this transform is [2]

$$T(x,\ \tau)=\left[-\exp\left(-\frac{2R-x-\Delta}{k\Delta}+\frac{4a_1\tau}{\Delta^2k^2}\right)\right]\operatorname{erfc}\left[\frac{1}{2}\cdot\frac{R-x-\Delta}{\sqrt{a_1\tau_1}}-\frac{2\sqrt{a_1\tau_1}}{\Delta k}\right]\frac{Q_1}{c_1\gamma_1k\Delta}, \tag{10}$$

or for x = −Δ

$$T(-\Delta,\ \tau)=T(0,\ \tau)=\left[-\exp\left(\frac{2R_1}{k\Delta}+\frac{4a_1\tau}{\Delta^2k^2}\right)\right]$$

$$\times\operatorname{erfc}\left[\frac{1}{2}\cdot\frac{R_1}{\sqrt{a_1\tau}}-\frac{2\sqrt{a_1\tau}}{\Delta k}\right]\frac{Q_1}{c_1\gamma_1k\Delta}. \tag{11}$$

The series expansion of the error function

$$\operatorname{erfc} y=\frac{1}{\sqrt{\pi}}\left[\exp(-y^2)\right]\cdot\left[\frac{1}{y}-\frac{1}{2y^3}+\cdots\right]$$

yields, after a few minor transformations,

$$T(0,\ \tau)=\frac{Q_1}{2c_1\gamma_1}\cdot\frac{R}{\sqrt{\pi\,\mathrm{Fo}}}\exp\left(-\frac{1}{4\,\mathrm{Fo}}\right)\frac{1}{1-\frac{\Delta}{R}k\frac{1}{4\,\mathrm{Fo}}}$$

$$\times\left[1-\frac{1}{2\left(\frac{1}{2\sqrt{\mathrm{Fo}}}-\frac{2\sqrt{\mathrm{Fo}}\,R}{\Delta k}\right)^2}+\cdots\right]. \tag{12}$$

Taking into account Eq. (1), we have

$$T(-\Delta,\ \tau)=T_{\mathrm{un}}(R,\ \tau)\frac{4a_1\tau}{4a_1\tau-R\Delta k}\times$$

$$\times\left[1-\frac{1}{2\left(\frac{R}{\sqrt{2a_1\tau}}-\frac{2\sqrt{a_1\tau}}{\Delta k}\right)^2}+\cdots\right]. \tag{13}$$

Beginning with Fo ≈ Δk/R, one may disregard all terms in the series after the unity and the resulting error will not exceed Δk/10R. As a result, taking into account relation (1), we have

$$T(0,\ \tau)=T_{\mathrm{un}}(0,\ \tau)\frac{1}{1-\frac{\Delta k}{R\,4\,\mathrm{Fo}}}$$

or for Fo ≈ 0.5 one may expand the fraction into a series

$$T(0,\ \tau) = T_{\text{un}}(0,\ \tau)\left(1 + \frac{\Delta}{R} k \frac{1}{4\,\text{Fo}}\right),$$

i.e.,

$$\delta T = \frac{T_{\text{un}}(0,\ \tau) - T(0,\ \tau)}{T_{\text{un}}(0,\ \tau)} = -k\frac{\Delta}{R} \cdot \frac{1}{4\,\text{Fo}},$$

$$\delta T_{\max} = -k\frac{1}{2}\ \frac{\Delta}{R}.$$

With the aid of (12), one can find $\tau_{\max}$ when the probe is in. Differentiating (12) and then equating to zero yields, after a few simple transformations

$$\tau_{\max} = \tau_{\max_{\text{un}}}\left(1 - k\frac{\Delta}{R}\right). \tag{16}$$

The distortion of the thermal flux is estimated in an analogous manner. For this, we find

$$\delta Q = \frac{Q_{\text{un}}(-\Delta,\ \tau) - Q(-\Delta,\ \tau)}{Q_{\text{un}}(-\Delta,\ \tau)}. \tag{17}$$

Here, starting from (1) and (10) with all second-order terms disregarded, we obtain

$$Q_{\text{un}}(-\Delta,\ \tau) = -\lambda\left(\frac{dT_{6.c}}{dx}\right)_{-\Delta}$$

$$= \frac{Q_1}{2c_1\gamma_1\sqrt{\pi\,\text{Fo}}\,R}\exp\left[-\frac{1}{4\,\text{Fo}}\right]\left(1 - \frac{\Delta}{R} \cdot \frac{1}{2\,\text{Fo}}\right)\frac{1}{2\,\text{Fo}\,R}\left(1 + \frac{\Delta}{R}\right), \tag{18}$$

$$Q(-\Delta,\ \tau) = -\lambda\left(\frac{dT_3}{dx}\right)_{-\Delta} \cong \frac{Q_1}{2c_1\gamma_1\sqrt{\pi\,\text{Fo}}\,R}$$

$$\times \exp\left(-\frac{1}{4\,\text{Fo}}\right)\frac{1}{\left(1 - \frac{\Delta}{R} \cdot \frac{k}{4\,\text{Fo}}\right)^2}\left\{\frac{1}{2\,\text{Fo}\,R}\left[1 - \frac{\Delta k}{4\,\text{Fo}\,R} - \frac{\Delta k}{R}\right]\right\} \tag{19}$$

and, after a few simple transformations,

$$\delta Q = -\frac{\Delta}{R}\left(\frac{k}{2\,\text{Fo}} - 1\right) \tag{20}$$

or

$$\delta Q_{\max} = -\frac{\Delta}{R}(k - 1). \tag{21}$$

The final expressions (14), (16), and (20) yield an estimate of the error in the measured values of the parameters. Both δT and δQ are uniquely determined by the probe dimensions and by the ratio k of specific heats. Since the probe readings of temperature are too high, hence an experiment yields higher values of the measured properties (thermal conductivity and specific heat) because

$$\lambda(R) = \frac{Q_1}{T_{\max}\tau_{\max}\sqrt{2\pi e}},$$

$$c\gamma(R) = \frac{Q_1}{T_{\max} R\sqrt{2\pi e}}.$$

It is possible to minimize the error in the readings of temperature and thermal flux by a proper choice of the ratio Δ/R, or, if that is technically not feasible, one may use the proper correction according to formulas (14), (16), and (20).

NOTATION

λ_1 is the thermal conductivity;
$c_1\gamma_1$ is the specific heat referred to volume;
T is the temperature;
T_{un} is the temperature in the medium under undistorted conditions;

Q_1 is the quantity of heat generated per unit area;
R is the distance from the heat source;
Δ is the probe thickness;
k is the ratio of specific heats, specimen material to probe material;
τ is the time;
τ_{max} is the instant of time which corresponds to the maximum temperature on the heating curve;
Q_{un} is the thermal flux in the medium under undistorted conditions;
Q is the thermal flux with the probe in;
T_{max} is the temperature at time τ_{max}.

LITERATURE CITED

1. A. F. Chudnovskii, Thermophysical Properties of Disperse Materials [in Russian], Fizmatgiz (1962).
2. Tables of Integral Transformations [in Russian], Izd. Nauka, Moscow (1969), Vol. 1.